Let $C \subset \mathbb{P}^2$ be a curve given by $f = 0$ where $f \in S = \mathbb{C}[x, y, z]$. Denote by $J_f$ the Jacobian ideal of $f$, i.e. the ideal generated by the partial derivatives of $f$. We describe the relations between the Milnor algebra $M(f) = S/J_f$ of $f$ and the singularities of $C$ which can be done by a study of the cohomology of the Koszul complex of the partial derivatives of $f$. We also give a description of the Hodge filtration on the cohomology groups $H^*(U)$ of the complement $U = \mathbb{P}^3 \setminus C$ of $C$. 